

Role of Partially Conserved Axial-Vector Current in the $\pi_{\mu 2}$ and $K_{\mu 2}$ Decays^{*)}

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§1. Introduction

Cabibbo's theory of leptonic decays of strongly interacting particles seems to be consistent with the present experimental data. In this theory, the strangeness-changing components in both vector and axial-vector currents are related by a single parameter called the Cabibbo angle.¹⁾ As for the dynamical origin of this angle, there are at present two distinct points of view: one is to assume that the angle is a manifestation of the effects of the symmetry breaking interaction which is responsible for the observed mass spectra.²⁾ The other is to assume that the angle is not related to the symmetry-breaking interaction and its origin is in the fundamental structure of weak interactions.

In the case of vector currents, it was shown that the Cabibbo angle does not receive renormalization corrections to first order in the symmetry breaking interactions.³⁾ This result may be taken as an indication which favors the latter point of view, that is, the origin of the angle is not in the symmetry-breaking interaction. However, the situation is not at all clear in the case of axial-vector currents.^{3), 4)} First of all, the current is not conserved even in the exact symmetry limit. This indicates that the renormalization effect does not necessarily vanish and that the effect could be very large.

In spite of these theoretical uncertainties, it is quite remarkable that the Cabibbo angle for the axial-vector currents (deduced from the $\pi_{\mu 2}$ and $K_{\mu 2}$ decays) is almost equal to that of the vector currents (deduced from the $\pi_{e 3}$ and $K_{e 3}$ decays). It is also remarkable that the F/D ratio obtained by Cabibbo for the axial-vector weak leptonic interactions is nearly equal to that of the strong baryon- PS -meson interaction which has been estimated independently by several authors.⁵⁾ Thus we are naturally led to ask whether there is a mechanism similar to that of the conserved vector currents in the case of axial-vector current. Probably the easiest way to understand these features of the axial-vector interaction is to assume that

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a generalization of the Goldberger-Treiman relation (which seems to be well established in the case of nucleon-lepton interaction) to all baryon-lepton interactions is valid.⁶⁾ This type of argument, however, requires *very accurate* $SU(3)$ relations for the strong baryon- PS -meson coupling constants.

In determining the axial-vector Cabibbo angle, it was assumed that the amplitudes for the $\pi_{\mu 2}$ and $K_{\mu 2}$ decays are insensitive to their mass variables. We point out, however, that this is not obvious since the mass difference between the pion and the K -meson is one of the most outstanding manifestations of the $SU(3)$ symmetry breaking interaction. In this paper we first re-examine this point. Using once-subtracted dispersion relations, we evaluate the decay amplitudes when the mass variables are reduced to the $SU(3)$ degenerate value. It is pointed out that if the relation derived from the partially conserved axial-vector current is drastically violated *it is quite possible that the Cabibbo picture could be very different from the present form*. However, a more important observation in this paper will be the fact that *the present value of the Cabibbo angle can be maintained even if we allow a moderate (and reasonable) amount of symmetry breaking effects*. We shall show in the following sections that the angle can be stable (within 10%) even if the generalized Goldberger-Treiman relations are violated to a certain extent, that is, the weak F/D ratio could be considerably different from the strong F/D ratio without modifying the present Cabibbo picture. This observation may particularly be useful if, in the future, the more precise measurements indicate that the generalized Goldberger-Treiman relations do not hold accurately.

We then make an attempt to see whether the origin of the Cabibbo angle is in the symmetry breaking interaction by constructing a dynamical model in which the symmetry breaking interaction directly affects the mass spectra whereas all other quantities receive the effect only through the mass spectra. Our overall feeling is that the Cabibbo angle is not related to the symmetry breaking interaction.

§2. Formulation of the problem

We consider first the $\pi^+ \rightarrow \mu^+ + \nu'$ decay. The leptons are assumed to emerge from a point. The matrix element depends only on the four-momentum of the π^+ meson. The "S" matrix element for this transition is proportional to M_{fi}^π :

$$M_{fi}^\pi = \bar{u}(p_\mu) i\gamma_\lambda (1 + \gamma_5) u(p_{\nu'}) \langle 0 | A_\lambda^{(0)} | \pi \rangle \quad (1)$$

where $A_\lambda^{(0)}$ is the strangeness-conserving part of the axial-vector current

operator. p_μ and $p_{\nu'}$ are respectively the four-momenta of the muon and the neutrino. We write the matrix element $\langle 0|A_\lambda^{(0)}|\pi\rangle$ in the form

$$\langle 0|A_\lambda^{(0)}|\pi\rangle = -i(p_\pi)_\lambda F_\pi(-p_\pi^2)/(2p_{\pi 0})^{1/2}. \quad (2)$$

Similarly, we introduce the matrix element $\langle 0|A_\lambda^{(1)}|K\rangle$ for the $K^+ \rightarrow \mu^+ + \nu'$ decay.

$$\langle 0|A_\lambda^{(1)}|K\rangle = -i(p_K)_\lambda F_K(-p_K^2)/(2p_{K0})^{1/2}, \quad (3)$$

where $A_\lambda^{(1)}$ is the strangeness non-conserving axial-vector current. We assume that *both* $A_\lambda^{(0)}$ and $A_\lambda^{(1)}$ transform predominantly as members of the $SU(3)$ octet.

We note that the amplitude $F_\pi(-p_\pi^2)$ and $F_K(-p_K^2)$ are functions of the π -meson and K -meson masses respectively. In order that the Cabibbo's procedure be valid these mass dependences must be negligible. In the following discussions, we examine this point and stress the role of the partially conserved axial-vector current hypothesis.

It was shown by various authors that both $F_\pi(-p_\pi^2)$ and $F_K(-p_K^2)$ satisfy dispersion relations in their arguments.⁷⁾ We write here once-subtracted dispersion relations,

$$\begin{aligned} F_\pi(s) &= f_\pi + \frac{(s - \mu_0^2)}{\pi} \int_{\mu_\pi^2}^{\infty} \frac{ds' \operatorname{Im} F_\pi(s')}{(s' - \mu_0^2)(s' - s)}, \\ F_K(s) &= f_K + \frac{(s - \mu_0^2)}{\pi} \int_{(\mu_K + 2\mu_\pi)^2}^{\infty} \frac{ds' \operatorname{Im} F_K(s')}{(s' - \mu_0^2)(s' - s)}, \end{aligned} \quad (4)$$

and choose the common subtraction point μ_0^2 to be the (mass)² of the $SU(3)$ degenerate pseudoscalar mesons, which is the arithmetic average of those of the π and η mesons,

$$\mu_0^2 = \frac{1}{2} (\mu_\pi^2 + \mu_\eta^2). \quad (5)$$

Depending on the mechanism of symmetry breaking interaction, the value of the degenerate mass μ_0 may be different from the one assumed above. However, our numerical results will not be sensitive to the value of μ_0^2 as long as μ_0^2 stays in the interval (which is reasonable),

$$\mu_\pi^2 \leq \mu_0^2 \leq \mu_\eta^2.$$

We note in Eq. (4) that when $(-p_\pi^2) \rightarrow \mu_0^2$ and $(-p_K^2) \rightarrow \mu_0^2$,

$$F_\pi(\mu_0^2) = f_\pi,$$

$$F_K(\mu_0^2) = f_K.$$

By choosing the common subtraction point μ_0^2 , we calculate first the

decay amplitudes when both the pion and K -meson masses are reduced to the $SU(3)$ degenerate value. The difference between the subtraction constant and the observed amplitude will give a criterion on the validity of Cabibbo's assumption. Furthermore, we shall argue in a later section that the subtraction constants could be regarded, to a rough approximation, as the decay amplitudes in the absence of the symmetry breaking interaction.

Now the observed decay amplitudes are

$$\begin{aligned} F_\pi(\mu_\pi^2) &= f_\pi \left[\frac{(\mu_\pi^2 - \mu_0^2)}{\pi} \int \frac{ds' \operatorname{Im} F_\pi(s')}{(s' - \mu_0^2)(s' - \mu_\pi^2)} \right], \\ F_K(\mu_K^2) &= f_K \left[\frac{(\mu_K^2 - \mu_0^2)}{\pi} \int \frac{ds' \operatorname{Im} F_K(s')}{(s' - \mu_0^2)(s' - \mu_K^2)} \right]. \end{aligned} \quad (7)$$

Since the value of μ_0^2 is greater than μ_π^2 and smaller than μ_K^2 , the factors $(\mu_\pi^2 - \mu_0^2)$ and $(\mu_K^2 - \mu_0^2)$ in the above expression have opposite signs while the dispersion integrals are expected to have the same sign. Thus we expect that the term containing the dispersion integral would give a positive contribution to one decay amplitude and a negative contribution to the other.

In the following sections we shall first evaluate the subtraction constants and then discuss a model in which the quantities f_π and f_K are respectively the strangeness-conserving and strangeness violating components of the $SU(3)$ symmetric decay amplitudes in the limit where the symmetry-breaking interaction which is responsible for the observed mass spectra is turned off.

§3. Evaluation of integrals and numerical results

In order to evaluate the dispersion integrals of Eq. (4) we first use the unitarity condition to express $\operatorname{Im} F_\pi(s)$ and $\operatorname{Im} F_K(s)$ in terms of known quantities. The lowest-mass intermediate state which can contribute to $\operatorname{Im} F_\pi(s)$ is the three-pion intermediate state. The strength of this contribution is estimated in the Appendix and is shown to be small. It is argued there that we can safely restrict our attention only to the two-body intermediate state.

The lowest-mass two-body state for $\operatorname{Im} F_\pi(s)$ is that of ρ and π mesons. But we postpone the discussion of this state to a later part of this section and take up first the baryon-antibaryon intermediate state. For the reasons which will be explained in the next section, we shall ignore here the small mass differences for baryons and use the $SU(3)$ degenerate baryon mass. Now, in order to calculate $\operatorname{Im} F_\pi(s)$, we consider the nucleon-antinucleon intermediate state. For the amplitude $\pi \rightarrow N\bar{N}$, we use the

perturbation amplitude corresponding to Fig. 1a.^{*)} In the amplitude $N\bar{N} \rightarrow$ leptons, we are led to consider the Feynmann amplitudes corresponding to the diagrams of Fig. 1b.



Fig. 1a. Nucleon-antinucleon contribution through the four-fermion axial-vector interaction.

1b. Nucleon-antinucleon contribution through $\pi \rightarrow \mu + \nu$ interaction.

After a standard algebra, we obtain the contribution of this $N\bar{N}$ intermediate state to the absorptive part⁷⁾

$$\text{Im} F_{\pi}^{N\bar{N}}(s) = -\frac{G}{8\pi} \left\{ 2mg^A + \frac{GF_{\pi}(\mu_{\pi}^2)s}{s - \mu_{\pi}^2} \right\} \left(\frac{s - 4\mu_{\pi}^2}{s} \right)^{1/2}, \quad (8)$$

where m is the degenerate baryon mass. g^A is the $N\bar{N} \rightarrow$ leptons axial-vector coupling constant. G is the π - N coupling constant. Before completing the dispersion integral we have to consider the $\bar{\Lambda}\Sigma$ and other baryon-antibaryon intermediate states. Now in the approximation of setting $(\mu_{\pi}/2m)^2 \rightarrow 0$, we simply obtain

$$F_{\pi}(\mu_{\pi}^2) = f_{\pi} - \frac{(\mu_{\pi}^2 - \mu_0^2)}{48\pi^2 m^2} \sum_{\lambda} G_{\lambda} \{ 2mg^A + G_{\lambda} F_{\pi}(\mu_{\pi}^2) \},$$

where λ denotes the nucleon-antinucleon and other baryon-antibaryon intermediate states which should be included in the $SU(3)$ scheme. The strong interaction coupling constants G_{λ} are calculated from the π - $\bar{N}N$ coupling constant, Clebsch-Gordan coefficients and the F/D ratio.^{**)} For the nucleon-antinucleon intermediate state, the Goldberger-Treiman relation

$$2mg^A \simeq -GF_{\pi}(\mu_{\pi}^2) \quad (9)$$

holds experimentally, which is valid up to its sign within a 5% error. As we shall point out later, the only theoretical basis for the above relation is the hypothesis of partially conserved axial-vector currents. Assuming that the situation is similar in all other intermediate states and summing up over λ , we arrive at the conclusion

^{*)} The structure of the π - $N\bar{N}$ vertex was studied in reference 7). But the structure of the type discussed there leads effectively to a smaller $\pi N\bar{N}$ coupling constant. Since we are concerned here with only upper limits of the $N\bar{N}$ contribution, the use of Feynman amplitudes is justified.

^{**)} For instance, we took here a value $F/D \simeq 1/2 \sim 2/3$. It turns out in later discussions that the only conditions on the F/D ratios (weak and strong separately) are the inequalities of Eqs. (11) and (15').

$$F_\pi(\mu_\pi^2) = \frac{f_\pi}{1 + \varepsilon_\pi}, \quad \text{where } |\varepsilon_\pi| < 0.03. \quad (10)$$

In arriving at this numerical result, we used $G^2/4\pi \simeq 15$.

We notice here that the decay amplitude is extremely stable against the variation of the mass variable. As we shall discuss in detail for the case of the K_{μ_2} decay, $F_\pi(\mu_\pi^2)$ and f_π are very close even if a possible violation of the Goldberger-Treiman relation is allowed. It is easy to show that ε_π is less than 15 percent for

$$-0.2|G_\lambda^0| < G_\lambda < 1.2|G_\lambda^0|, \quad (11)$$

where G_λ^0 satisfies the exact Goldberger-Treiman relation

$$2mg_\lambda^A = -G_\lambda^0 F_\pi(\mu_\pi^2).$$

Let us next consider $F_K(\mu_K^2)$. In this case we are led to consider first the $N\bar{A}$ intermediate state, whose contribution to $\text{Im} F_K(s)$ is

$$\text{Im} F_K^{N\bar{A}}(s) = -\frac{G'}{8\pi} \left(2mg^{A'} + \frac{G' F_K(\mu_K^2) s}{s - \mu_K^2} \right) \left(\frac{s - 4\mu_K^2}{s} \right)^{1/2},$$

where $g^{A'}$ is the $\bar{A}N$ -lepton axial-vector coupling constant and is related to g^A by the Cabibbo angle. G' is the $KN\bar{A}$ coupling constant. After summing up all possible intermediate baryon-antibaryon states we obtain

$$F_K(\mu_K^2) = f_K - \frac{(\mu_K^2 - \mu_0^2)}{48\pi^2 m^2} \sum_\lambda G'_\lambda \{ 2mg_\lambda^{A'} + G'_\lambda F_K(\mu_K^2) \}, \quad (12)$$

where λ again denotes all possible intermediate states. Unlike the case of nucleon-antinucleon state for $F_\pi(\mu_\pi^2)$ there is so far neither experimental evidence nor a theoretical justification for the Goldberger-Treiman type relation for the $N\bar{A}$ or other intermediate states under consideration.

If the extended Goldberger-Treiman relation holds:

$$2mg_\lambda^{A'} = -G'_\lambda F_K(\mu_K^2), \quad (13)$$

we simply obtain

$$F_K(\mu_K^2) \simeq f_K.$$

In order that the Goldberger-Treiman relation of Eq. (13) hold, *both the strong and weak interactions must have the same F/D ratio*. We shall see, however, that the Cabibbo angle remains stable even if the Goldberger-Treiman relation is violated to a considerable extent. Let us write the value of G'_λ which satisfies Eq. (13) as $G_\lambda^{0'}$, i.e.

$$2mg_\lambda^{A'} = -G_\lambda^{0'} F_K(\mu_K^2). \quad (14)$$

We note here that the quantities $F_K(\mu_K^2)$ and $g_\lambda^{A'}$ (for instance, in the $A \rightarrow p + e + \bar{\nu}$ decay) are measured experimentally. It is indicated that the value of $G_\lambda^{0'}$ ($K\bar{A}p$ coupling constant) is as strong as is required by the $SU(3)$ symmetry. On the other hand, there is an independent experimental indication from the γ - K production that the K -coupling may be smaller than that predicted by the unitary symmetry. In order to take these facts into account, we first assume without loss of generality that $G_\lambda^{0'}$ is positive. If we confine $G_\lambda^{0'}$ in the region

$$0 \leq G_\lambda^{0'} \leq G_\lambda^{0'0}, \quad (15)$$

then the contribution of the $N\bar{A}$ state to $F_K(\mu_K^2)$ in Eq. (12) takes its maximum value when

$$G_\lambda^{0'} = \frac{1}{2} G_\lambda^{0'0}.$$

If Eq. (15) holds for all other channels under consideration, we arrive at the conclusion

$$F_K(\mu_K^2) = \frac{f_K}{1 + \epsilon_K}, \quad \text{where } |\epsilon_K| \leq 0.06. \quad (16)$$

It is easy to show that the above result still holds for $G_\lambda^{0'}$ in the extended region

$$0 + \frac{1 - \sqrt{2}}{2} G_0^{0'} \leq G_\lambda^{0'} \leq \frac{1 + \sqrt{2}}{2} G_0^{0'}$$

or

$$0.2G_0^{0'} \leq G_\lambda^{0'} \leq 1.2G_0^{0'}. \quad (15')$$

From the above analysis, we see that $F_K(\mu_K^2)$ and f_K can differ by at most 6%. Of course, if the extended Goldberger-Treiman relation, like Eq. (14), holds for every intermediate state, ϵ_K of Eq. (16) will be much smaller than 0.06 and $F_K(\mu_K^2)$ will be much closer to f_K . This requires the same F/D ratio for both strong and weak couplings and, therefore, strong K -baryon couplings as are required by the unitary symmetry. The above argument, however, indicates that even though the Goldberger-Treiman relation is violated for the strangeness-changing currents, $F_K(\mu_K^2)$ and f_K are nearly equal as long as $G_\lambda^{0'}$ lies in the region of Eq. (15').

We have seen above that both the strangeness-conserving and strangeness-changing decay amplitudes receive a very small contribution from the baryon-antibaryon intermediate states, if the deviation from the extended Goldberger-Treiman relation is in the range given by Eq. (15') and its counterpart for the $\pi_{\mu 2}$ decay. This means that the F/D ratios of the

strong and weak interactions can be considerably different without requiring a large change of the value of the Cabibbo angle. This may explain the rather unexpected success of Cabibbo's theory for the case of axial-vector part.⁶⁾

As for other two-body intermediate states, such as pseudoscalar-vector meson and baryon- N^* states which are equally important (for instance, in the $SU(6)$ theory), we first note that they contribute to the absorptive part as a higher partial wave. Furthermore, we expect that a cancellation quite similar to that of the baryon-antibaryon case occurs also in these intermediate states. Thus the strangeness-conserving and strangeness-changing decay amplitudes are very close to the subtraction constants f_π and f_κ respectively.

In order to stress the importance of the role of the sign in the Goldberger-Treiman relation, we repeat here the calculation with the opposite sign, that is, with the positive sign on the right-hand side of Eq. (9). Assuming that the situation is the same for all other intermediate states for $F_\pi(\mu_\pi^2)$, we derive

$$F_\pi(\mu_\pi^2) \simeq 3f_\pi. \quad (17)$$

Assuming, next, that the exact Goldberger-Treiman relations with the opposite sign for the $K \rightarrow \mu + \nu'$ decay with the intermediate states considered before, i.e. $2mg^{A'} = +G^0 F_\kappa(\mu_\kappa^2)$, we obtain the relation

$$F_\kappa(\mu_\kappa^2) \simeq \frac{3}{4} f_\kappa. \quad (18)$$

Then from Eqs. (17) and (18) we see that the equality (zero Cabibbo angle)

$$f_\pi \approx f_\kappa$$

can give the observed value of the Cabibbo angle, that is

$$F_\pi(\mu_\pi^2) \simeq 4F_\kappa(\mu_\kappa^2).$$

Although we do not attach any significance to the above number, this result points out the importance of the role of the sign in the Goldberger-Treiman relation. As for the choice of the sign in question, we mention first that the measurement of the $\pi \rightarrow \mu + \nu'$ decay rate does not determine the sign. The only concrete experimental evidence which favors the usual sign is the μ -capture experiment. As for the original derivation of the relation by Goldberger and Treiman, we point out that they use an unsubtracted dispersion relation and therefore their result is not related to the present work. The only theoretical basis for the usual sign is the hypothesis of partially conserved axial-vector current⁸⁾ which has recently

been shown to be consistent with the π - N scattering data.⁹⁾ Thus we conclude that the hypothesis of partially conserved axial-vector current leads to a very small deviation of the decay amplitude from the subtraction constant in both $\pi \rightarrow \mu + \nu'$ and $K \rightarrow \mu + \nu'$ decays.*¹⁾ In the following section, we shall discuss a dynamical model in which the subtraction constants f_π and f_K are respectively the strangeness-conserving and the strangeness-changing decay amplitudes when the interaction responsible for the observed mass spectrum is switched off.

§4. Concluding remarks

It was shown in the preceding section that the hypothesis of the partially conserved axial-vector current leads to a very small difference between the observed decay amplitude and the subtraction constant at the degenerate PS -meson mass value. In this section, we consider a dynamical model in which the subtraction constant at the degenerate mass can be regarded as the decay amplitude in the absence of the symmetry breaking interaction.

We note first of all that the baryon mass differences were ignored throughout the preceding calculation and that our result will not be significantly modified even if they are taken into account. The PS -meson mass difference, especially that between the π and K mesons is relatively a large quantity, and its effect on others has been the subject of the present investigation. We have, in fact, calculated the difference $(F_\pi(\mu_\pi^2) - f_\pi)$ in terms of the mass difference $(\mu_\pi^2 - \mu_0^2)$. In view of the fact that the symmetry breaking interaction is much more prominent in the PS -meson masses, we are led to consider a dynamical model where the departure from the exact symmetry of the quantities such as the decay amplitude can be achieved by an analytic continuation in the PS -meson mass variable only.

In the preceding sections it was seen that only particles playing significant roles are the baryons and the pseudoscalar mesons. In the system of these two particles, we first write the $SU(3)$ -invariant Lagrangian with the degenerate masses for both sets of particles. In order to take into account the symmetry breaking effect, we then add a term which is not invariant under the $SU(3)$ transformation. Let us consider for concreteness the symmetry breaking interaction of the ω - φ mixing type.¹⁰⁾ By conservation of the isospin, hyper-charge and baryonic number, the vector currents of the ω and φ mesons couple to baryons and the PS -mesons with an *equal* strength. On the ground that the net effect of this interaction on the baryon mass is relatively small and that the PS -meson part

*¹⁾ See the remark at the end of §4.

alone will give at least a correct order of magnitude in the final answer, we ignore the baryonic part of the symmetry breaking interaction. Then the symmetry breaking interaction appears only in the mass of the pseudoscalar mesons. This effectively leads us to construct a model in which the departure from the exact symmetry is achieved by an analytic continuation in the PS -meson mass variables from the degenerate value.

We have observed in the preceding section that the dispersion integrals such as

$$\int_{\mu_\pi^2}^{\infty} \frac{ds' \operatorname{Im} F_\pi(s')}{(s' - \mu_0^2)(s' - \mu_\pi^2)}$$

of Eq. (4) are insensitive to the PS -meson mass value. The only significant mass dependence comes from the dispersion variable. Thus in this crude model, the subtraction constant f_π can be regarded as the decay amplitude in the absence of the symmetry breaking interaction if the constants we used in evaluating the dispersion integrals are insensitive to the symmetry-breaking effect. In the preceding discussion, we have in fact estimated the effect of the variation of the strong PS -meson-baryon coupling constant, and our quantitative conclusions took this effect into account. Thus, if the symmetry breaking interaction does not cause a wider variation than we have considered, the subtraction constants can be regarded as the decay amplitudes in the absence of the symmetry breaking interaction. Our feeling is, therefore, that the origin of the Cabibbo angle is not in the symmetry breaking interaction responsible for the observed mass spectra.

Several interesting interpretations of the origin of this angle have been proposed.¹¹⁾ Now, since the contribution of the PS -meson pole terms to the ratio of leptonic decays of baryons are small and since, in our picture, the effect of baryon mass differences is relatively unimportant, we expect that the overall effect of the symmetry breaking interaction is small in the case of leptonic decays of baryons. However, we have shown in the preceding discussions that this effect is small also in the case of $\pi_{\mu 2}$ and $K_{\mu 2}$ decays provided that the relations based on the partially conserved axial-vector current are valid to a certain extent. Therefore, if this is the case, we can expect that the value of the angle determined from the $\pi_{\mu 2}$ and $K_{\mu 2}$ decays will, more or less (say within 10 percent), agree with that from the leptonic decays of baryons. The present experimental data seem to support this possibility.¹²⁾ It should be emphasized, however, that this does not require the *very accurate* validity of the Goldberger-Treiman relations. That is, we may maintain the Cabibbo picture in the presence of the symmetry breaking interaction even if its effect turns out to be more important in the case of the $SU(3)$ invariant strong interaction.

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Appendix

In this appendix we estimate the strength of the three pion intermediate state for $\text{Im}F_\pi(s)$. In order to estimate the contribution from this state, we take perturbation theoretic amplitudes for both the $\pi \rightarrow 3\pi$ and $3\pi \rightarrow \text{leptons}$ as illustrated in Fig. 2. Then the contribution to $\text{Im}F_\pi(s)$ is

$$\text{Im}F_\pi^{(3\pi)}(s) = 64(4\pi\lambda)^2 \left(\frac{1}{2\pi}\right)^5 F_\pi(\mu_\pi^2) \int \frac{d^3k_1 d^3k_2 d^3k_3}{8k_{10}k_{20}k_{30}} \delta(k_1+k_2+k_3-p_\pi),$$

where k_1 , k_2 and k_3 are the four momenta of the three intermediate pions. The coupling constant λ is defined from the four-pion Lagrangian

$$\mathcal{L}_{4\pi} = 4\pi\lambda(\boldsymbol{\pi} \cdot \boldsymbol{\pi} + 2\bar{K}K + 2\bar{K}^0 K^0 + \eta^2)^2.$$

We evaluate the three-body unitarity integral in the frame where $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$ and show after a straight-forward algebra that the contribution to the dispersion integral in Eq. (4) is

$$\alpha_{3\pi}^{3\pi} = 64 \frac{(\mu_\pi^2 - \mu_0^2)}{\mu_\pi^2} \left(\frac{\lambda^2}{4\pi}\right) \frac{\mu_\pi^2}{\pi} F_\pi(\mu_\pi^2) \int_{\mu_\pi}^{\infty} \frac{dW J(W)}{W(W^2 - \mu_0^2)(W^2 - \mu_\pi^2)^2},$$

where

$$J(W) = \int_{4\mu_\pi^2}^{(W-\mu_\pi)^2} dx \left\{ \frac{(x-4\mu_\pi^2)}{x} \left| x - 2(W^2 + \mu_\pi^2) + (W^2 - \mu_\pi^2)^2 \right| \right\}^{1/2}$$

In the above integral, the factor $J(W)$ makes a significant contribution only in the region $W > 5\mu_\pi$ while the rest of the integrand does in the region $W < 5\mu_\pi$. As a consequence, the numerical value of the integral is very small:

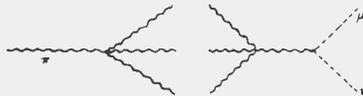


Fig. 2. Three-pion contribution to $\text{Im}F_\pi(s)$.

$$\alpha_{\pi}^{3\pi} \simeq 64 \left(\frac{\lambda^2}{4\pi} \right) \left(\frac{\mu_{\pi}^2 - \mu_0^2}{\mu_{\pi}^2} \right) F_{\pi}(\mu_{\pi}^2) \times 2 \times 10^{-8}.$$

According to the best estimation

$$\lambda^2 \simeq 0.02.$$

Thus

$$\alpha_{\pi}^{3\pi} \simeq -2F_{\pi}(\mu_{\pi}^2) \times 10^{-8}.$$

The contribution from the three-pion intermediate state considered here is less than one percent of the observed decay amplitude. We therefore ignore this intermediate state. Although it is much more complicated to make any meaningful calculations for other multiparticle states, we expect that the contributions from these states are similarly suppressed. We expect furthermore that the situation is similar in the $K\pi\pi$ and other intermediate states for the use of $\text{Im}F_{\kappa}(s)$. This result allows us to restrict our attention only to two-body intermediate states.

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