

Standing Waves in the Lorentz-Covariant World

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When Einstein formulated his special relativity, he developed his dynamics for point particles. Of course, many valiant efforts have been made to extend his relativity to rigid bodies, but this subject is forgotten in history. This is largely because of the emergence of quantum mechanics with wave-particle duality. Instead of Lorentz-boosting rigid bodies, we now boost waves and have to deal with Lorentz transformations of waves. We now have some understanding of plane waves or running waves in the covariant picture, but we do not yet have a clear picture of standing waves. In this report, we show that there is one set of standing waves which can be Lorentz-transformed while being consistent with all physical principle of quantum mechanics and relativity. It is possible to construct a representation of the Poincaré group using harmonic oscillator wave functions satisfying space-time boundary conditions. This set of wave functions is capable of explaining the quantum bound state for both slow and fast hadrons. In particular it can explain the quark model for hadrons at rest, and Feynman's parton model hadrons moving with a speed close to that of light.

KEY WORDS: Lorentz covariance; standing waves; bound states; relativistic quantum mechanics.

1. INTRODUCTION

Einstein formulated his special relativity 100 years ago while making Newtonian mechanics consistent with the Lorentz-covariant world. In so doing, he derived his energy-momentum relation valid for both massive and massless particles. Einstein of course formulated his theory for point particles. Since then, there have been efforts to understand special relativity for rigid particles with non-zero size, without any tangible results. On

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the other hand, the emergence of quantum mechanics made the rigid-body problem largely irrelevant.

Because of the wave-particle duality, quantum mechanics is sometimes called wave mechanics. Instead of rigid bodies, we talk about wave packets and standing waves. The issue becomes whether those waves can be made Lorentz-covariant.

Of course, here, the starting point is the plane wave, which can be written as

$$e^{ip \cdot x} = e^{i(\vec{p} \cdot \vec{x} - Et)}. \quad (1)$$

Since it takes the same form for all Lorentz frames, we do not need any extra effort to make it covariant.

Indeed, the S-matrix derivable from the present form of quantum field theory calls for calculation of all S-matrix quantities in terms of plane waves. Thus, the S-matrix is associated with perturbation theory or Feynman diagrams. Indeed, Feynman propagators are written in terms of plane waves on the mass shell.

We should realize however that the S-matrix formalism is strictly for running waves, starting from a plane wave from one end of the universe and ending with another plane wave at another end. How about standing waves? This question is illustrated in Fig. 1. Of course, standing waves can be regraded as superpositions of running waves moving in opposite directions. However, in order to guarantee localization of the standing waves, we need a spectral function or boundary conditions. The covariance of standing waves necessarily involve the covariance of boundary conditions or spectral functions. How much do we know about this problem?

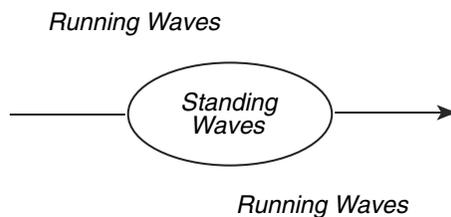


Fig. 1. Running waves and waves in quantum theory. If a particle is allowed to travel from infinity to infinity, it corresponds to a running wave according to the wave picture of quantum mechanics. If, on the other hand, it is trapped in a localized region, we have to use standing waves to interpret its location in terms of probability distribution.

The purpose of this paper is to examine this problem systematically. When we talk about standing waves in quantum mechanics, we start with two standard examples, namely harmonic oscillators and particles bound by hard walls separated by a space-like distance. For the hard walls, we do not know how to deal with the covariance of the boundary conditions, and we are not able to report anything in this paper.

For harmonic oscillators, boundary conditions are smooth, it might be possible to impose a localization condition in a Lorentz-covariant manner. This possibility was considered by a number of great physicists in the past, including Dirac,⁽¹⁾ Yukawa,⁽²⁾ and Feynman *et al.*⁽³⁾ The paper of Feynman *et al* was written after Gell-Mann's formulation of the quark model,⁽⁴⁾ and is much closer to the real world.

Therefore, in this report, we start with the Lorentz-invariant differential equation given by Feynman *et al.*⁽³⁾ Our first step is to make up mathematical deficiencies of this paper, and to construct a set of covariant harmonic oscillator wave functions. We then attach physical interpretations to these wave functions. We point out that the covariant oscillator formalism satisfies all the known rules of quantum mechanics and special relativity, as the present form of quantum field theory does.

In addition, we point out that the covariant oscillator formalism can explain the quark model for hadrons when they are at rest or slow, and that the same formalism leads to Feynman's parton model when they move with speed close to that of light. Indeed, the quark model and the parton model are two limiting cases of one covariant entity.

In Sec. 2, it is noted that there are running waves and standing waves in quantum mechanics. While it is easy to Lorentz-boost running waves, it requires covariance of boundary conditions to understand fully standing waves. In Sec. 3, we discuss the space-time symmetry applicable to standing waves in the Lorentz-covariant regime. It is pointed out that this symmetry is dictated by Wigner's little group^(5,6) for massive particles.

In Sec. 4, it is shown possible to construct a set of harmonic oscillator wave functions, which can be Lorentz-boosted. It is shown that these wave functions are compatible with all known rules of quantum mechanics and special relativity. As a physical application of this covariant harmonic oscillator formalism, it is shown in Sec. 5 that the quark and parton models are two different manifestation of the same covariant entity. In Sec. 6, we present a future roadmap for quantum mechanics in the Lorentz-covariant world.

2. SCATTERING STATES AND BOUND STATES

We are now facing the problem of whether the basic concept of quantum mechanics survives in Einstein's Lorentzian world. By now, it is safe to assume that Feynman diagrams serve our purpose well for scattering states. Feynman diagrams are possible because the covariant form for plane waves is quite trivial.

How about bound states? In order to understand the bound-state problem, we have to understand standing waves in the covariant world as indicated in Fig. 1. In his talk presented at the 1970 April meeting of the American physical society held in Washington, DC, Feynman stunned the audience by saying that Feynman diagrams are not applicable to bound state problems.^(7,8) He suggested harmonic oscillators for a possible solution.

We can summarize what Feynman said in Fig. 2. Feynman's point was that, while plane-wave approximations in terms of Feynman diagrams work well for relativistic scattering problems, they are not applicable to bound-state problems. For bound-state problems, we should perhaps try harmonic oscillator wave functions. Feynman's 1970 talk was later published in the paper of Feynman *et al.* in the Physical Review.⁽³⁾

Although this paper contained the above mentioned original idea of Feynman, it contains serious mathematical flaws. Feynman *et al.* start with a Lorentz-invariant differential equation for the harmonic oscillator for the quarks bound together inside a hadron. For the two-quark system, they write the wave function of the form

$$\exp \left\{ -\frac{1}{2} (z^2 - t^2) \right\}, \quad (2)$$

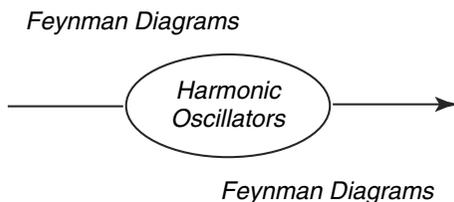


Fig. 2. Feynman's roadmap for combining quantum mechanics with special relativity. Feynman diagrams work for running waves, and they provide a satisfactory resolution for scattering states in Einstein's world. For standing waves trapped inside an extended hadron, Feynman suggested harmonic oscillators as the first step.

where z and t are the longitudinal and time-like separations between the quarks. This form is invariant under the boost, but is not normalizable in the t variable.

On the other hand, the Gaussian form

$$\exp \left\{ -\frac{1}{2} (z^2 + t^2) \right\} \quad (3)$$

also satisfies Feynman's Lorentz-invariant differential equation. This Gaussian function is normalizable, but is not invariant under the boost. However, the word "invariant" is quite different from the word "covariant." The above form can be covariant under Lorentz transformations. We shall get back to this problem in Sec. 4.

Feynman *et al.* studied in detail the degeneracy of the three-dimensional harmonic oscillators, and compared with the observed experimental data. Their work is complete and thorough, and is consistent with the $O(3)$ -like symmetry dictated by Wigner's little group for massive particles.^(5,6) Yet, Feynman *et al.* make an apology that the symmetry is not $O(3, 1)$. This unnecessary apology causes a confusion not only to the readers but also to the authors themselves, and makes the paper difficult to read.

3. SPACE-TIME SYMMETRY OF STANDING WAVES

As was noted in Sec. 2, it is trivial to Lorentz-transform plane waves. How about superposition of plane waves? We have to deal with the Lorentz covariance of their spectral functions. This is not an easy problem for standing waves consisting of waves moving in opposite directions. We shall come back to this problem in Sec. 4. In this section, we shall study the space-time symmetry of standing waves.

In the Lorentz-covariant world, the word standing wave means that there is at least one Lorentz frame in which the amplitude is non-zero in a localized spacial region, and this localization region stays at the same place independent of time. We shall call this Lorentz frame "the rest frame" for the standing wave. How would this standing wave look to an observer moving with a constant velocity? We can safely say the whole system will move with a constant velocity in the opposite direction. How about the shape of the standing wave? Is the concept of localization preserved under Lorentz boosts?

In order to tackle this problem, we have to understand the space-time symmetry of this localized system. The Lorentz group applicable to a free particle has six parameters corresponding to three rotations around and

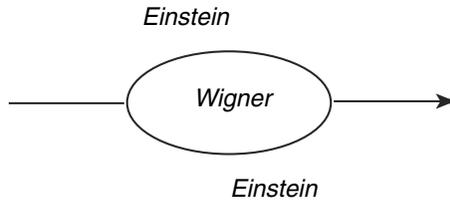


Fig. 3. Wigner in Einstein's world. Einstein formulates special relativity whose energy-momentum relation is valid for point particles as well as particles with internal space-time structure. It was Wigner who formulated the framework for internal space-time symmetries by introducing his little groups whose transformations leave the four-momentum of a given particle invariant.

three boosts along the three orthogonal spatial directions. Once the system is given in a specific Lorentz frame, the system has only three degrees of freedom, as is seen in Wigner's 1939 paper on his little groups.⁽⁵⁾ The system regains all of the six degrees of freedom when we add the three degrees of freedom to boost in three independent directions.⁽⁹⁾ Indeed, the bound state or the standing wave has only three rotational space-time degrees of freedom in the Lorentz frame in which it is at rest. This picture of space-time symmetry is illustrated in Fig. 3.

This aspect of Wigner's little group has already been studied in the past. Let us see where the present problem stands in the development of this subject.

Since Einstein introduced the Lorentz covariant space-time symmetry, his energy momentum relation $E = \sqrt{p^2 + m^2}$ has been proven to be valid for not only point particles, but also particles with internal space-time structure defined by quantum mechanics. Particles can have quantized spins if they are at rest or they are slowly moving. If, on the other hand, the particle is massless and moves with speed of light, it has its helicity which is the spin parallel to its momentum and gauge degree of freedom.

Table I summarizes the covariant picture of the present particle world. The second row of this table indicates that the spin symmetry of slow particles and the helicity-gauge symmetry of massless particles are two limiting cases of one covariant entity called Wigner's little group. This issue has been extensively discussed in the literature.⁽¹⁰⁾

Let us then concentrate on the third row of Table I. After Einstein formulated his special relativity, a pressing problem was to see whether his relativistic dynamics can be extended to rigid bodies as in the case of Newton's sun and earth and their rotations. Their rotations are translated

Table I. Massive and massless particles in one package

	Massive, Slow	Covariance	Massless, Fast
Energy–Momentum Internal Space–time Symmetry	$E = p^2/2m$ S_3 S_1, S_2	Einstein’s $E = [p^2 + m^2]^{1/2}$ Wigner’s Little Group	$E = p$ S_3 Gauge Trans
Relativistic Extended Particles	Quark Model	One Covariant Theory	Parton Model

Wigner’s little group unifies the internal space-time symmetries for massive and massless particles. It is a great challenge for us to find another unification: the unification of the quark and parton pictures in high-energy physics.

into particle spins in quantum mechanics. Their sizes can be translated into the width of the standing waves. Is special relativity going to prevail for these standing waves?

As we pointed out in this section, Wigner’s formulation of the $O(3)$ -like little group was a very important step. We are extending this concept to standing waves. The development of the quark model for hadrons was another important step toward understanding Einstein’s covariance.⁽⁴⁾ The proton is a quantum bound state of quarks. Since the proton these days can achieve a velocity very close to that of light, it is a relativistic bound-state in the real world. While the proton is like a bound state when it is at rest, it appears as a collection of partons when it moves with velocity close to that of light. As we shall discuss in Sec. 5, partons have properties which appear to be quite different from those of quarks. Can we produce a standing wave solution for the proton which can explain both the quark model and the parton model? This is the problem defined in the third row of Table I.

4. CAN HARMONIC OSCILLATORS BE MADE COVARIANT?

As we emphasized in Sec. 2, quantum field theory has been quite successful in terms of perturbation techniques in quantum electrodynamics. However, this formalism is based on the S-matrix for scattering problems and useful only for physical processes where a set of free particles becomes another set of free particles after interaction. Quantum field theory does not address the question of localized probability distributions and their covariance under Lorentz transformations. The Schrödinger quantum mechanics of the hydrogen atom deals with localized probability

distribution. Indeed, the localization condition leads to the discrete energy spectrum. Here, the uncertainty relation is stated in terms of the spatial separation between the proton and the electron. If we believe in Lorentz covariance, there must also be a time-separation between the two constituent particles.

Before 1964,⁽⁴⁾ the hydrogen atom was used for illustrating bound states. These days, we use hadrons which are bound states of quarks. Let us use the simplest hadron consisting of two quarks bound together with an attractive force, and consider their space-time positions x_a and x_b , and use the variables

$$X = (x_a + x_b)/2 \quad x = (x_a - x_b)/2\sqrt{2}. \quad (4)$$

The four-vector X specifies where the hadron is located in space and time, while the variable x measures the space-time separation between the quarks. According to Einstein, this space-time separation contains a time-like component which actively participates as can be seen from

$$\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh \eta & \sinh \eta \\ \sinh \eta & \cosh \eta \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}, \quad (5)$$

when the hadron is boosted along the z direction. In terms of the light-cone variables defined as,⁽¹¹⁾

$$u = (z + t)/\sqrt{2} \quad v = (z - t)/\sqrt{2}, \quad (6)$$

the boost transformation of Eq. (5) takes the form

$$u' = e^\eta u \quad v' = e^{-\eta} v. \quad (7)$$

The u variable becomes expanded while the v variable becomes contracted, as is illustrated in Fig. 4.

Does this time-separation variable exist when the hadron is at rest? Yes, according to Einstein. In the present form of quantum mechanics, we pretend not to know anything about this variable. Indeed, this variable belongs to Feynman's rest of the universe.

Also in the present form of quantum mechanics, there is an uncertainty relation between the time and energy variables. However, there are no known time-like excitations. Unlike Heisenberg's uncertainty relation applicable to position and momentum, the time and energy separation variables are c -numbers, and we are not allowed to write down the commutation relation between them. Indeed, the time-energy uncertainty relation is a c -number uncertainty relation,⁽¹²⁾ as is illustrated in Fig. 5.

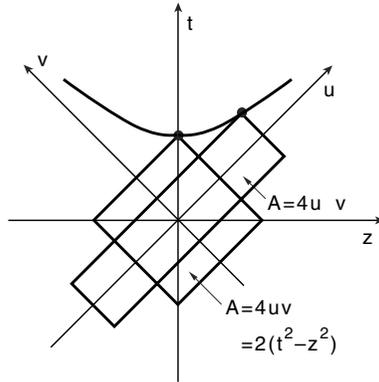


Fig. 4. Lorentz boost in the light-cone coordinate system.

How does this space-time asymmetry fit into the world of covariance.⁽¹³⁾ This question was studied in depth by the present authors in the past. The answer is that Wigner’s $O(3)$ -like little group is not a Lorentz-invariant symmetry, but is a covariant symmetry.⁽⁵⁾ It has been shown that the time–energy uncertainty applicable to the time–separation variable fits perfectly into the $O(3)$ -like symmetry of massive relativistic particles.⁽⁶⁾

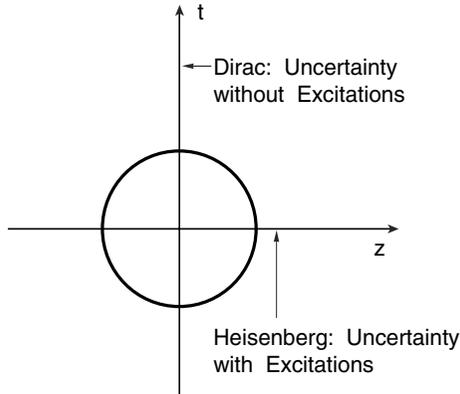


Fig. 5. Space-time picture of quantum mechanics. There are quantum excitations along the space-like longitudinal direction, but there are no excitations along the time-like direction. The time-energy relation is a c-number uncertainty relation.

The c -number time-energy uncertainty relation allows us to write down a time distribution function without excitations.⁽⁶⁾ If we use Gaussian forms for both space and time distributions, we can start with the expression

$$\left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(z^2 + t^2)\right\} \quad (8)$$

for the ground-state wave function. What do Feynman *et al.* say about this oscillator wave function?

In their classic 1971 paper,⁽³⁾ Feynman *et al.* start with the following Lorentz-invariant differential equation.

$$\frac{1}{2} \left\{ x_\mu^2 - \frac{\partial^2}{\partial x_\mu^2} \right\} \psi(x) = \lambda \psi(x). \quad (9)$$

This partial differential equation has many different solutions depending on the choice of separable variables and boundary conditions. Feynman *et al.* insist on Lorentz-invariant solutions which are not normalizable. On the other hand, if we insist on normalization, the ground-state wave function takes the form of Eq. (8). It is then possible to construct a representation of the Poincaré group from the solutions of the above differential equation.⁽⁶⁾ If the system is boosted, the wave function becomes

$$\psi_\eta(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(e^{-2\eta}u^2 + e^{2\eta}v^2)\right\}. \quad (10)$$

This wave function becomes Eq. (8) if η becomes zero. The transition from Eqs. (8) to (10) is a squeeze transformation. The wave function of Eq. (8) is distributed within a circular region in the uv plane, and thus in the zt plane. On the other hand, the wave function of Eq. (10) is distributed in an elliptic region with the light-cone axes as the major and minor axes, respectively. If η becomes very large, the wave function becomes concentrated along one of the light-cone axes. Indeed, the form given in Eq. (10) is a Lorentz-squeezed wave function. This squeeze mechanism is illustrated in Fig. 6.

There are many different solutions of the Lorentz invariant differential equation of Eq. (9). The solution given in Eq. (10) is not Lorentz invariant but is covariant. It is normalizable in the t variable, as well as in the space-separation variable z . How can we extract probability interpretation from this covariant wave function?

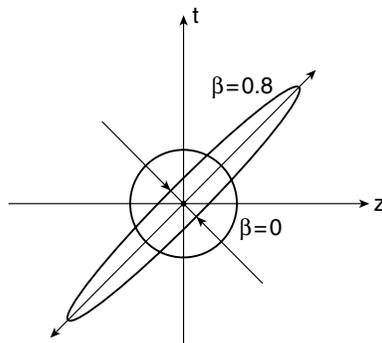


Fig. 6. Effect of the Lorentz boost on the space-time wave function. The circular space-time distribution in the rest frame becomes Lorentz-squeezed to become an elliptic distribution.

5. FEYNMAN'S PARTON PICTURE

It is a widely accepted view that hadrons are quantum bound states of quarks having localized probability distribution. As in all bound-state cases, this localization condition is responsible for the existence of discrete mass spectra. The most convincing evidence for this bound-state picture is the hadronic mass spectra which are observed in high-energy laboratories.^(3,6)

In 1969, Feynman observed that a fast-moving hadron can be regarded as a collection of many "partons" whose properties appear to be quite different from those of the quarks.⁽¹⁴⁾ For example, the number of quarks inside a static proton is three, while the number of partons in a rapidly moving proton appears to be infinite. The question then is how the proton looking like a bound state of quarks to one observer can appear different to an observer in a different Lorentz frame? Feynman made the following systematic observations.

- (a) The picture is valid only for hadrons moving with velocity close to that of light.
- (b) The interaction time between the quarks becomes dilated, and partons behave as free independent particles.
- (c) The momentum distribution of partons becomes widespread as the hadron moves fast.

- (d) The number of partons seems to be infinite or much larger than that of quarks.

Because the hadron is believed to be a bound state of two or three quarks, each of the above phenomena appears as a paradox, particularly (b) and (c) together.

In order to resolve this paradox, let us write down the momentum-energy wave function corresponding to Eq. (10). If we let the quarks have the four-momenta p_a and p_b , it is possible to construct two independent four-momentum variables⁽³⁾

$$P = p_a + p_b \quad q = \sqrt{2}(p_a - p_b), \quad (11)$$

where P is the total four-momentum. It is thus the hadronic four-momentum.

The variable q measures the four-momentum separation between the quarks. Their light-cone variables are

$$q_u = (q_0 - q_z)/\sqrt{2} \quad q_v = (q_0 + q_z)/\sqrt{2}. \quad (12)$$

The resulting momentum-energy wave function is

$$\phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}\left(e^{-2\eta}q_u^2 + e^{2\eta}q_v^2\right)\right\}. \quad (13)$$

Because we are using here the harmonic oscillator, the mathematical form of the above momentum-energy wave function is identical to that of the space-time wave function. The Lorentz squeeze properties of these wave functions are also the same. This aspect of the squeeze has been exhaustively discussed in the literature.^(6,16)

When the hadron is at rest with $\eta = 0$, both wave functions behave like those for the static bound state of quarks. As η increases, the wave functions become continuously squeezed until they become concentrated along their respective positive light-cone axes. Let us look at the z -axis projection of the space-time wave function. Indeed, the width of the quark distribution increases as the hadronic speed approaches that of the speed of light. The position of each quark appears widespread to the observer in the laboratory frame, and the quarks appear like free particles.

The momentum-energy wave function is just like the space-time wave function, as is shown in Fig. 7. The longitudinal momentum distribution becomes wide-spread as the hadronic speed approaches the velocity of light. This is in contradiction with our expectation from non-relativistic quantum

QUARKS → PARTONS

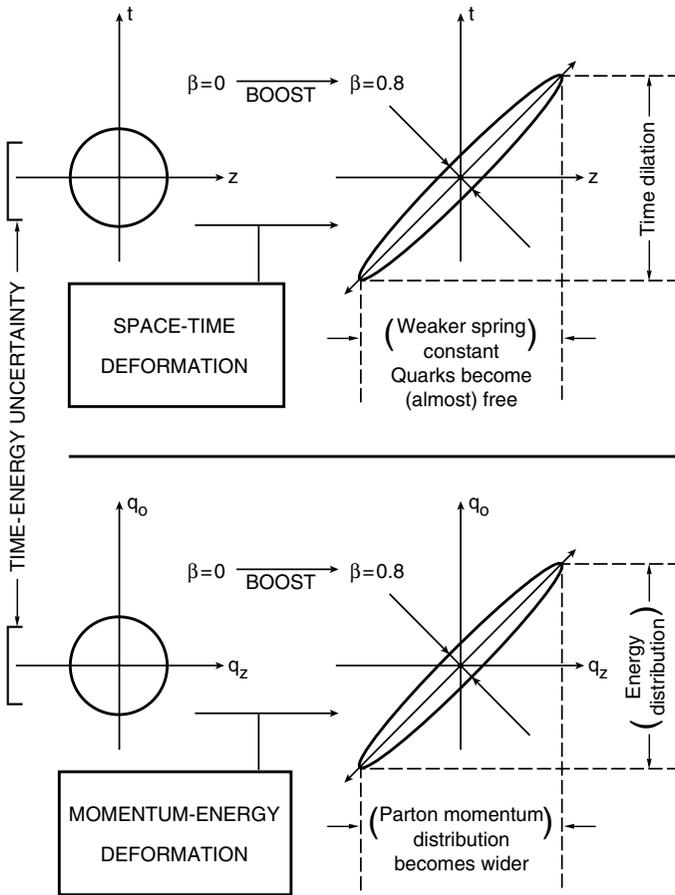


Fig. 7. Lorentz-squeezed space-time and momentum-energy wave functions. As the hadron's speed approaches that of light, both wave functions become concentrated along their respective positive light-cone axes. These light-cone concentrations lead to Feynman's parton picture.

mechanics that the width of the momentum distribution is inversely proportional to that of the position wave function. Our expectation is that if the quarks are free, they must have their sharply defined momenta, not a wide-spread distribution.

However, according to our Lorentz-squeezed space-time and momentum-energy wave functions, the space-time width and the

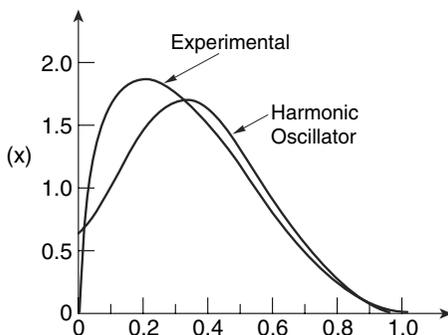


Fig. 8. Parton distribution function. Theory and experiment.

momentum-energy width increase in the same direction as the hadron is boosted. This is of course an effect of Lorentz covariance. This indeed is the key to the resolution of the quark-parton paradox.^(6,15)

After these qualitative arguments, we are interested in whether Lorentz-boosted bound-state wave functions in the hadronic rest frame could lead to parton distribution functions. If we start with the ground-state Gaussian wave function for the three-quark wave function for the proton, the parton distribution function appears as Gaussian as is indicated in Fig. 8. This Gaussian form is compared with experimental distribution also in Fig. 8.

For large x region, the agreement is excellent, but the agreement is not satisfactory for small values of x . In this region, there is a complication called the “sea quarks.” However, good sea-quark physics starts from good valence-quark physics. Figure 8 indicates that the boosted ground-state wave function provides a good valence-quark physics.

Feynman’s parton picture is one of the most controversial models proposed in the 20th century. The original model is valid only in Lorentz frames where the initial proton moves with infinite momentum. It is gratifying to note that this model can be produced as a limiting case of one covariant model which produces the quark model in the frame where the proton is at rest. We need Feynman’s parton model to complete the third row of Table I.

6. HISTORY, FUTURE, AND STRINGS

In this paper, we are dealing with two different histories. One is how to deal with relativistic extended particles starting from Einstein’s special relativity for point particles, as illustrated in Table I. In so doing, we

Table II. History of scattering states and bound states

	Scattering	Unified	Bound States
Before Newton	Comets	Unknown	Planets
Newton	Hyperbola	Newton	Ellipse
Quantum Mechanics	Running Waves	Particle Waves	Standing Waves
Feynman	Diagrams	Unknown	Oscillators
Future Theory	Running Waves *Fields	One Physics	Standing Waves *Strings

The history starts with open and closed orbits of astronomical objects. Newton unified the elliptic and hyperbolic orbits with his Newtonian mechanics. In quantum mechanics with wave-particle duality, running waves and standing waves tell the difference between bound states and scattering states. The remaining problem is whether this quantum picture remains valid in Einstein's covariant world.

had to face another historical problem, namely the problem of whether scattering problem and bound-state problem can be treated by the same dynamics, as shown in Table II. This history starts from the ancient mystery that comets and planets have different orbits.

Historically, the unified picture of scattering and bound states was accomplished by an invention of new dynamics. As we can see from Table II, the completion of Newtonian mechanics was accompanied by a unified view of elliptical and hyperbolic orbits.

At the beginning of the 20th century, discrete energy levels emerged as one of the most pressing puzzling problems in physics. Why do bound states have discrete energy levels while scattering states do not. This question was solved by the wave-particle duality of quantum mechanics, where bound states satisfy localization boundary condition.

In the world of Einstein, the scattering problem is now well understood in terms of quantum field theory and Feynman diagrams. In this paper, we studied whether the covariant harmonic oscillator formalism could serve as a model for relativistic bound states. We strengthened our earlier assertion that it satisfies every known physical principle as quantum field theory does.⁽¹⁷⁾ In this report, we discussed the same problem with the space-time symmetry of standing waves in the framework of Wigner's little group for massive particles.

We are of course aiming at a unified covariant theory which will take care of both scattering and bound-state problems. In the case of the Schrödinger quantum mechanics, we start with one differential equation, and the difference comes from the boundary condition dictated by localization of probability distribution.

In the covariant world of quantum mechanics, the story is the same. For free particles, the Lorentz-invariant Klein-Gordon equation is the starting point. The covariant oscillator formalism starts also with a Lorentz-invariant differential equation. The main difference between running and standing waves is in the boundary conditions, as in the case of the Schrödinger quantum mechanics. In field theory, we talk about asymptotic conditions where particles are free particles in the remote past and remote future in the Lorentz-covariant world, where causality is preserved. In the oscillator formalism, we talk about localization boundary conditions in the Lorentz-covariant world.

Finally, let us make a comment on current activities in string theory. The ultimate purpose of string theories is to understand space-time symmetry of particles with internal structures. As we pointed out in Sec. 3, this symmetry was worked out by Wigner in his fundamental paper of 1939. His work is totally consistent with Einstein's covariant world.

In addition, in string theories, there are internal vibrations within particles. For vibrational problems, we are not aware of any simpler model than harmonic oscillators. Let us keep in mind that, in both engineering and science, it is customary to reduce all complicated vibrational problems into simple harmonic oscillators before making contacts with the real world. Therefore, the most urgent problem in string theories is to reduce the problem to a soluble model, namely the covariant harmonic oscillator formalism presented in this report.

This paper indeed provides a place for string theory in the roadmap of relativity and quantum mechanics, as illustrated in Table II.

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