Physical Principles in Quantum Field Theory and in Covariant Harmonic Oscillator Formalism

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Abstract

It is shown that both covariant harmonic oscillator formalism and quantum field theory are based on common physical principles which include Poincare covariance, Heisenberg's space-momentum uncertainty relation, and Dirac's "Cnumber" time-energy uncertainty relation. It is shown in particular that the oscillator wave functions are derivable from the physical principles which are used in the derivation of the Klein-Nishina formula.

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1 Introduction

Ever since the development of quantum mechanics, its reconciliation with the principle of relativity has been the most important problem in theoretical physics. The earliest attempt to combine these two theories was made by Schrodinger even before the present form of quantum mechanics was formulated [1]. Since then there has been progress, but this goal has not yet been achieved. The purpose of this paper is to point out that present-day research in high-energy experimental and theoretical physics is moving toward this goal. There are now enough experimental data and enough theoretical models to enable us to think deeply about the possible underlying physical principles which are consistent with both quantum mechanics and relativity.

The most important step taken in this direction is of course the covariant form of quantum field theory. However, it has its well-known limitations. Field theory can explain some scattering processes where all initial and final-state particles are free particles. However, it is not yet clear whether field theory can provide answers to bound-state problems where the concept of localized probability plays the central role [2, 3]. Opinion is divided at the present time. While some physicists believe that we will eventually find the answers purely within the framework of field theory algorithms, there are also others who believe that this goal will never be realized.

The present authors approached this problem by first constructing relativistic bound-state wave functions even though they appeared, mathematically, quite different from those expected in quantum field theory. Our criterion was simply to construct at least one set of bound-state wave functions consistent with the established principles of quantum mechanics and relativity, which could at the same time explain basic high-energy hadronic phenomena [4]. The second step in our approach should then be to see whether the physical principles employed in constructing such wave functions are identical to those for quantum field theory. The purpose of the present paper is to show that the answer to this question is "Yes."

Figure 1 summarizes what we have done in the past and what we intend to do in the future. In the past, we restricted our attention to the physics of confined particles using the harmonic oscillator model. This work belongs to "Step 1" in Fig. 1. In the present paper, we are making an attempt to find the set of physical principles which are applicable to both field theory and oscillator formalism. We propose to start doing "Step 2."

In Section 2, we examine the present status of quantum field theory and

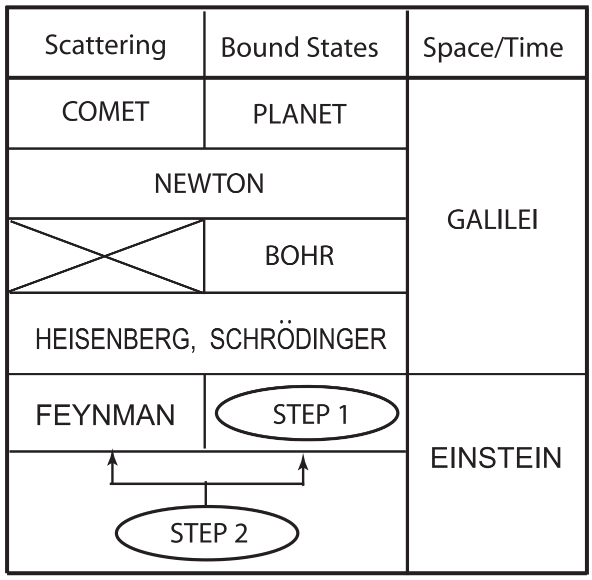

Figure 1: History of dynamical and kinematical developments. It is important to note that mankind's unified understanding of scattering and bound states has been very brief. It is, therefore, not unusual to expect that separate theoretical models be developed for scattering and for bound states. The successes and limitations of the Feynman diagram approach are well known. The covariant harmonic oscillator formalism belongs to Step I. In order that both field theory and the oscillator formalism be useful for constructing a theoretical model belonging to Step 2, they should be based on the same set of physical principles.

that of the oscillator formalism, and then spell out our line of attack. In Section 3, we examine the physical principles in quantum field theory, using the so-called "old-fashioned" field theory. Section 4 contains a discussion of the physical principles upon which the covariant oscillator formalism is based. It is shown that field theory and oscillator formalism, although they take quite different mathematical forms, are based on the same set of physical principles. In Section 5, it is pointed out that both field theory and oscillator model are needed to describe consistently covariantly relativistic extended hadrons at the present time, in the absence of a single mathematical formalism which can describe both the scattering and bound/confined states. In Section 6, we discuss deeper physical implications of the comparative study presented in this paper.

2 Formulation of the Problem

As is specified in Fig. 1, quantum field theory primarily deals with particles whose asymptotic states are those of free particles. The field theory algorithm allows us to describe creation and annihilation of particles which we observe in the real world. On the other hand, field theory is not effective in describing bound-state problems as is manifested in the present wellknown difficulty associated with atempts to describe confined quarks within the field theory framework [5], as well as in the well-established calculations in quantum electrodynamics.

The wave functions in the covariant oscillator model, while being consistent with the known rules of quantum mechanics and relativity, can explain all the basic features of relativistic hadrons [4]. However, the model can only describe the quarks which are permanently confined. From this fact alone, it is not at all clear whether the spacetime behavior of the wave functions describable in the oscillator model has anything to do with that of point particles whose asymptotic states are free-particle states. In view of both the advantages and disadvantages mentioned above, it would be ideal if we could construct a relativistic theory which contains the advantage of field theory applicable to scattering states and the advantage of the oscillator formalism applicable to bound or confined particles. Because the mathematical algorithm of field theory is so different from that of the oscillator formalism, it does not seem to be realistic at this time to attempt to construct a single mathematical apparatus which will produce both of the advantages mentioned above. However, we can still ask the following two questions.

1. Can we construct a field theory of extended hadrons where the standard Feynman rules are applicable to hadrons with free-particle asymptotic states, while the oscillator-like formalism is used to describe the internal motion of confined quarks? There are already several approaches based on this idea [6, 7].

2. Although quantum field theory and the covariant oscillator formalism take different mathematical forms, it is possible that they are based on the same physical principles. If so, what are the physical principles underneath these two different-looking theories? The purpose of the present paper is to discuss the second question. For this purpose, we note first that both formalisms are covariant under Poincaré transformation. While the Feynman propagator and the wave function are the primary mathematical devices in field theory and the oscillator formalism, respectively, both quantities contain time-energy uncertainty in addition to Heisenberg's space-momentum uncertainty relation. We note further that both formalisms start with relativistic wave equations with negative energy spectra, and that both have their own respective ways of taking care of them. We shall discuss these points more systematically in the following sections.

3 Physical Principles in Quantum Field Theory

Field theory starts with the physical principles applicable to nonrelativistic quantum mechanics including of course Heisenberg's space-momentum uncertainty relation. The question is then what additional physical principles are used in quantum field theory.

First of all, field theory equations are relativistic. Thus, the Lorentz covariance is one of the basic additional ingredients, as is well known.

Next, let us look into the question of causality. The present form of field theory starts with the causal commutator

$$[\phi(x_1), \phi(x_2)] = i\Delta(x_1 - x_2), \tag{1}$$

where $\phi(x)$ is the field operator satisfying the Klein-Gordon equation. This commutation relation corresponds to Heisenberg's uncertainty relation when

$x_{10} = x_{20}$. The causal Green's function $\Delta(x_1 - x_2)$ vanishes outside the light cone:

$$\Delta(x_1 - x_2) = 0, \quad for \quad (x_1 - x_2)^2 < 0, \tag{2}$$

This means that the signal connecting two space-time points cannot propagate faster than light. In carrying out field theory calculations, however, we use more often the Feynman propagation function whose mathematical form is

$$\Delta(x_{-}x_2) = \left(\frac{1}{2\pi}\right)^4 \int d^4k \frac{\exp(-ik \cdot x)]}{k^2 - m^2 + i\epsilon}. \tag{3}$$

This function does not vanish outside the light cone. The reason for this is that the particle is no longer on the mass shell, and this unobservable particle does not respect causality. We are then naturally led to ask what additional physical laws are needed to explain this causality violation.

The derivation of the Feynman propagation function requires time and normal orderings which require "trading" ground-state energies with vacuum. In order to see the basic physics involved in this procedure more clearly, we now resort to the so-called "old-fashioned" field theory. Since all the basic physical concepts in the present covariant form of quantum field theory are contained in the "old-fashioned" field theory, it is not uncommon to refer to this earlier formalism in order to find explanations for what we do in the modern version of field theory [8].

For the purpose of finding the physical principles which led to the microscropic violation of causality and to the concept of "virtual" off-massshell particles, let us look at Chapter IV, Section 14 of Heitler's book [8]. The "old-fashioned" derivation of the Klein-Nishina formula is based on secondorder time-dependent pertubation theory involving two integrations over the time variable. The second time integration is done "correctly" from the interval 0 to t. However, the first time integration contains a causality violation for the period allowed by the "C-number" time–energy uncertainty relation. [4] This introduction of time–energy uncertainty leads to the concept of virtual particles.

In the modern version of quantum field theory, the time-energy uncertainty manifests itself in the off-mass-shell particles contained in Feynman

[4]The concept of the "C-number" time-energy uncertainty relation was introduced first by Dirac [10]. The word "C-number" means that there are no excitations (no motions in the classical sense) along the time-like direction [4, 11].

propagators. Although there is no precise definition of bound-state conditions in field theory [2, 3, 4], it is by now a widely accepted view that particles in bound states, which are affected by interactions, are not on their mass shells. In the following section, we shall see how this appears in the covariant oscillator model which deals only with quarks permanently bound inside a relativistic hadron.

4 Physical Principles in the Covariant Oscillator Formalism

The purpose of this section is to demonstrate that the covariant oscillator formalism employs the same physical principles as in quantum field theory, namely the Poincare covariance and the C-number time–energy uncertainty, in addition to Heisenberg's space-momentum uncertainty relation.

First, as we pointed out repeatedly in earlier papers, the concept of "C-number" time-energy uncertainty relation is one of the basic additional ingredients in the covariant oscillator formalism. For a hadron at rest consisting of two quarks bound together by a harmonic oscillator potential of unit strength, the wave function takes the form

$$\psi(z,t) = H_n(z) \exp\left[-\left(\frac{z^2 + +t^2}{2}\right)\right], \tag{4}$$

where z and t represent the longitudinal and time-like coordinate separations between the quarks. As was done frequently in earlier papers, we ignore here the transverse coordinates. The existence of the ground-state wave function in the time separation vatiable t (without excitations) represents Dirac's "Cnumber" time-energy uncertainty relation.

Second, the oscillator formalism, as in the case of quantum field theory, violates microscopic causality. The wave function given in Eq.(4) does not vanish for the space-like separation:

$$t^2 < z^2. \tag{5}$$

This causality violation is not unlike that needed in the "old-fashioned" derivation of the Klein-Nishina formula in second-order time-dependent perturbation theory [9].

Finally, as in the case of off-mass-shell particles in field theory, the bound-state quarks have unobservable masses. In order to see this point, let us go back to the covariant formalism of the oscillator model. Let x, and X2 denote the space-time coordinates for the first and second bound-state quarks, respectively. The usual procedure in handling this problem is to define the hadronic coordinate X:

$$X = \frac{x_1 + x_2}{2}, \tag{6}$$

and the relative quark separation coordinate x:

$$x = \frac{x_1 - x_2}{2\sqrt{2}}. \tag{7}$$

Then the Poincaré transformation is generated by the following generators:

$$P_\mu = i\frac{\partial}{\partial X^\mu}, \qquad M_{\mu\nu} = L^*_{\mu\nu} + L^*_{\mu\nu}, \tag{8}$$

where

$$L^*_{\mu\nu} = i\left(X_\mu \frac{\partial}{\partial X^\nu} - X_\nu \frac{\partial}{\partial X^\mu}\right),$$

$$L_{\mu\nu} = i\left(x_\mu \frac{\partial}{\partial x^\nu} - x_\nu \frac{\partial}{\partial x^\mu}\right).$$

The invariant Casimir operators in this case are

$$P^2 = P^\mu P_\mu, \quad and \quad W^2 = W^{mu} W_{mu}, \tag{9}$$

with

$$W_\mu = \left(\frac{1}{2}\right)\epsilon_{\mu\nu\alpha\beta} P^\nu W^{\alpha\beta}.$$

It has been shown that the essence of the oscillator formalism is to construct representations of the Poincare group which are diagonal in the Casimir operator. We noted further that such representations can be constructed from the wave functions of the form

$$\phi(X, x) = \psi(x, P)\exp\left(\pm P \cdot X\right), \tag{10}$$

where the "internal" wave function If/(x, P) satisfies the harmonic oscillator differential equation

$$H(x)\psi(x, P) = \lambda\psi(x, P), \tag{11}$$

with

$$H(x) = \frac{1}{2}\left[\left(\frac{\partial}{\partial x_\mu}\right)^2 - x_\mu^2\right]$$

subject to the subsidiary condition

$$P^\mu\left(x_\mu + \frac{\partial}{\partial x^\mu}\right)\psi(x, P) = 0. \tag{12}$$

P_μ is the four-momentum of the hadron. The $(mass)^2$ of the hadron P^2 is determined by the eigenvalues of the above oscillator differential equation:

$$P^2 = H(x) + m_0^2. \tag{13}$$

In order that any quantity be a Poincaré-invariant eigenvalue, it has to commute with the Casimir operators p2 and W2 It, therefore, has to commute with the above form of P', In order to see whether the mass of the bound-state quark is a Poincaré-invariant constant, let us write down the expressions for the quark four-momenta:

$$P_{1\mu} = \frac{i}{2}\left[\frac{\partial}{\partial X^\mu} + \left(\frac{1}{\sqrt{2}}\right)\frac{\partial}{\partial x^\mu}\right],$$

$$P_{2\mu} = \frac{i}{2}\left[\frac{\partial}{\partial X^\mu} - \left(\frac{1}{\sqrt{2}}\right)\frac{\partial}{\partial x^\mu}\right], \tag{14}$$

We can now calculate the p/ and p/, and take the commutators

$$[P_1^2, H(x)] = -\frac{1}{\sqrt{2}}\left[x_\mu\frac{\partial}{\partial x_\mu} + \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x_\mu} - 1\right)\right],$$

$$[P_2^2, H(x)] = \frac{1}{\sqrt{2}}\left[x_\mu\frac{\partial}{\partial x_\mu} + \frac{1}{\sqrt{2}}\left(\frac{\partial}{\partial x_\mu} - 1\right)\right]. \tag{15}$$

Since the above commutators do not vanish, the constituent masses cannot be regarded as Poincaré-invariant eigenvalues. In the language of quantum field theory, bound-state quarks are not on their mass shells, and this is a consequence of the time-energy uncertainty relation covariantly added to the existing rules of nonrelativistic quantum mechanics. If we use the language of the covariant oscillator formalism, the quark mass cannot be simultaneously diagonalized with the hadronic mass which is one of the Poincaré-invariant eigenvalues. This unobservable quark mass is indeed due to Dirac's "C-number" time-energy uncertainty relation which, together with the existing principles of nonrelativistic quantum mechanics, forms the physical basis for covariant harmonic oscillator formalism [4].

5 Further Field Theoretic Concepts in the Covariant Oscillator Model

Unlike the mass of the confined quark, the hadronic $(mass)^2$ corresponds to the eigenvalue of one of the Casimir operators of the Poincaré group [12]. The purpose of this section is to discuss the role of this Poincaré-invariant quantity in giving a field theoretic interpretation to the hadronic coordinate.

The covariant oscillator wave function of Eq.(10) is a solution of the following Poincaré-invariant differential equation [3].

$$\left\{2\left[\left(\frac{1}{\partial x_{1\mu}}\right)^2 + \left(\frac{1}{\partial x_{2\mu}}\right)^2\right] - \frac{1}{16}\left(x_1 - x_2\right)^2 + m_0^2\right\}\phi\left(x_1, x_2\right) = 0. \qquad (16)$$

If we use the variables X and x given in Eqs.(6) and (7), then the above equation is separable, and the solution takes the form given in Eq.(10). $\psi(x, P)$ satisfies the harmonic oscillator differential equation of Eq.(11), and the exponential factor $exp(iP \cdot X)$ satisfies the Klein-Gordon equation

$$\left[\left(1/\partial X_\mu\right)^2 + m_0^2 + \lambda\right]\exp\left(\pm iP \cdot X\right) = 0. \qquad (17)$$

It is important to note that the above free-hadron equation is also an integral part of the oscillator formalism. Both Eqs.(11) and (17) are relativistic equations, and give negative $(mass)^2$ and negative energy spectra, respectively. In the oscillator case, we have eliminated negative eigenvalues by imposing the subsidiary condition given in Eq.(12). As for negative energy spectra coming from the Klein-Gordon equation, we "take care" of them by giving the usual field theoretic interpretation. This is possible because the eigenvalue $(mass)^2$ of the oscillator equation is positive due to the subsidiary condition, and, therefore, the hadronic energy is always real.

We are, thus, led to the idea of giving an extended-particle interpretation to $\phi(X, x)$ of Eq.(10) by second-quantizing only the wave function associated with the hadronic coordinate X, and by leaving the relative coordinate x as a measure of the space-time distribution for the constituent quarks inside the extended hadron. It is interesting to note that there are already a number of papers in the literature on this subject [7].

6 Concluding Remarks

In developing new physical theories, there are, in general, two different approaches. According to Eddington, we have to understand all the physical principles before writing down the first formula. According to Dirac, it is more profitable to construct mathematical devices which can describe the real world, and then add physical interpretations to this formalism [13]. Both special relativity and quantum mechanics were developed in Dirac's way. In quantum field theory also, the mathematical development appears to precede the process of giving physical interpretations.

The Klein-Nishina formula and later calculations in quantum electrodynamics indeed represent the striking numerical successes of field theory. Field theory has been successful also in constructing a unified picture of weak and electromagnetic interactions [15]. It is regarded as the basic instrument for understanding the forces between the quarks, and there are numerous calculations based on this assumption [16]. On the conceptual front, the notion of virtual particles with unphysical masses is a product of the covariant formulation of field theory [5]. However, these successes do not remove all the unresolved conceptual difficulties. As was discussed extensively in the literature [2, 3, 5], the most serious difficulty in the present form of field theory is our inability to construct a localized space-time probability distribution which is so essential in interpreting relativistic bound states.

The covariant harmonic oscillator formalism was also developed first as a simple mathematical device which can explain measurable numbers in the relativistic quark and parton models [1, 14], and the task of attaching physical interpretations came later [4, 17]. This task of interpreting the oscillator formalism will not be complete until its relation to the present form of quantum field theory is clarified. We have shown in this paper that the covariant oscillators, while using the same physical principles as those in quantum field theory, make up the deficiency of field theory in explaining relativistic bound states.

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